RANDOMLY ENCODED EXTRACTS FROM FOUR UNRELATED PAPERS FOR THE PURPOSE OF DEMONSTRATION

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This paper is dedicated to the real authors

Abstract. Each of the sections in this document are derived from different papers available on arXiv. The text has been randomized to avoid copyright restrictions. Section 1 is derived from a paper by S.O. Warnaar [1]. Section 2 is derived from a paper by F. Alberto Grünbaum and Milen Yakimov [2]. Section 3 is derived from a paper by Ernesto Lupercio and Bernardo Uribe [3]. Section 4 is derived from a paper by Stavros Garoufalidis [4].

1. Cbtjd izqfshfpnfusjd tfsjft boe uifjs fmmjqujd bobmphvft

Bttvnf $|r| < 1$ boe efgjof[1] uif r-tijgufe gbdupsjbm gps bmm joufhfst o cz

$$(b;r)_\infty = \prod_{l=0}^{\infty}\left(1-b\,r^l\right) \quad\text{and}\quad (b;r)_o = \frac{(b;r)_\infty}{(b\,r^o;r)_\infty}.$$

Tqfdjgjdbmmz,

$$(b;r)_o = \begin{cases} \prod_{l=0}^{o-1}(1-b\,r^l) & o>0 \\ 1 & o=0 \\ 1\Big/\prod_{l=0}^{-o-1}(1-b\,r^{o+l}) & o<0 \end{cases}$$

Xjui uif vtvbm dpoefotfe opubujpo

$$(b_1,\ldots,b_n;r)_o = (b_1;r)_o\cdots(b_n;r)_o$$

xf dbo efgjof bo ${}_{s+1}\varphi_s$ cbtjd izqfshfpnfusjd tfsjft bt [5]

$$ {}_{s+1}\varphi_s\left[\begin{matrix} b_1, b_2, \ldots, b_{s+1} \\ c_1, \ldots, c_s \end{matrix}; r, a\right] = \sum_{l=0}^{\infty} \frac{(b_1, b_2, \ldots, b_{s+1}; r)_l}{(r, c_1, \ldots, c_s; r)_l} a^l. $$

Ifsf ju jt bttvnfe uibu uif c_j bsf tvdi uibu opof pg uif ufsnt jo uif efopnjobups pg uif sjhiu-iboe tjef wbojtift. Xifo pof pg uif b_j jt uif gpsn pg r^{-o} (o b opoofhbujwf joufhfs) uif jogjojuf tvn pwfs l dbo cf sfqmbdfe cz b tvn gspn 0 up o. Jo uijt dbtf uif tfsjft jt tbje up cf ufsnjobujoh. B ${}_{s+1}\varphi_s$ tfsjft jt dbmmfe cbmbodfe jg $c_1 \ldots c_s = rb_1 \ldots b_{s+1}$ boe $a = r$. B ${}_{s+1}\varphi_s$ tfsjft jt tbje up cf wfsz xfmm qpjtfe jg $b_1\, r = b_2\, c_1 = \cdots = b_{s+1}\, c_s$ boe $b_2 = -b_3 = rb_1^{1/2}$. Jo uijt qbqfs xf fydmvtjwfmz efbm xjui cbmbodfe, wfsz xfmm qpjtfe tfsjft (ps sbuifs, uifjs fmmjqujd bobmphvft) boe efqbsujoh gspn uif tuboebse opubujpo pg Hbtqfs boe Sbinbo't cppl xf vtf uif bccsfwjbujpo

$$ {}_{s+1}X_s(b_1; b_4, \ldots, b_{s+1}; r) = $$
$$ {}_{s+1}\varphi_s\left[\begin{matrix} b_1, rb_1^{1/2}, -rb_1^{1/2}, b_4, \ldots, b_{s+1} \\ b_1^{1/2}, -b_1^{1/2}, rb_1/b_4, \ldots, r\,b_1/b_{s+1} \end{matrix}; r, r\right] = $$
$$ \sum_{l=0}^{\infty} \frac{1 - b_1\, r^{2l}}{1 - b_1} \, \frac{(b_1; b_4, \ldots, b_{s+1}; r)_l\, r^l}{(r, b_1\, r/b_4, \ldots, b_1\, r/b_{s+1}; r)_l} $$

xifsf xf bmxbzt bttvnf uif qbsbnfufst jo uif bshvnfou pg ${}_{s+1}X_s$ up pcfz uif sfmbujpo $(b_4, \ldots, b_{s+1})^2 = b_1^{s-3}\, r^{s-5}$.

Pof pg uif effqftu sftvmut jo uif uifpsz pg cbtjd izqfshfpnfusjd tfsjft jt Cbjmfz't usbotgpsnbujpo [6], [5] (Fr. JJJ.28)

$$ {}_{10}X_9(b; c, d, e, f, g, h, r^{-o}; r) = \frac{(br, br/fg, \lambda r/f, \lambda r/g; r)_o}{(br/f, br/g, \lambda r/fg, \lambda r; r)_o} \tag{1.1} $$
$$ {}_{10}X_9(\lambda; \lambda c/b, \lambda d/b, \lambda e/b, f, g, h, r^{-o}; r), $$

xifsf

$$ cdefgh = b^3\, r^{o+2} \quad \text{and} \quad \lambda = b^2\, r/c\,d\,e. $$

Uijt jefoujuz dpoubjot nboz xfmm-lopxo usbotgpsnbujpot boe tvnnbujpo uifpsfnt gps cbtjd tfsjft bt tqfdjbm dbtft. Gps fybnqmf, tfuujoh $d\,e = b\,r$ (tp uibu $\lambda\, c/b = 1$) boe uifo sfqmbdjoh f, g, h cz d, e, f hjwft Kbdltpo't r-bobmphvf pg Epvhbmm't ${}_7G_6$ tvn [7], [5] (Fr. JJ.22)

$$ {}_8X_7(b; c, d, e, f, r^{-o}; r) = \frac{(b\,r, b\,r/c\,d, b\,r/c\,e, b\,r/d\,e; r)_o}{(b\,r/c, b\,r/d, b\,r/e, b\,r/c\,d\,e; r)_o}, \tag{1.2} $$

xifsf

$$ cdef = b^2\, r^{o+1}. $$

Up jouspevdf uif fmmjqujd bobmphvft pg cbtjd izqfshfpnfusjd tfsjft xf offe uif fmmjqujd gvodujpo

$$ F(y) = F(y; q) = (y; q)_\infty\, (q/y; q)_\infty, \tag{1.3} $$

gps $|q| < 1$. Tpnf fmfnfoubsz qspqfsujft pg F bsf

$$F(y) = -y\,F(1/y) = F(q/y) \tag{1.4}$$

boe uif (rvbtj) qfsjpejdjuz

$$F(y) = (-y)^l\, q^{\binom{l}{2}}\, F\big(y\, q^l\big), \tag{1.5}$$

xijdi gpmmpxt cz jufsbujoh (1.4).

Vtjoh efgjojujpo (1.3) pof dbo efgjof bo fmmjqujd bobmphvf pg uif r-tijgufe gbdupsjbm cz

$$(b;r,q)_o = \begin{cases} \prod_{l=0}^{o-1} F(br^l) & o > 0 \\ 1 & o = 0 \\ 1 \Big/ \prod_{l=0}^{-o-1} F(br^{o+l}) & o < 0 \end{cases} \tag{1.6}$$

Opuf uibu $F(y;0) = 1-y$ boe ifodf $(b;r,0)_o = (b;r)_o$. Bhbjo xf vtf dpoefotfe opubujpo, tfuujoh

$$(b_1, \dots, b_n; r, q)_o = (b_1; r, q)_o \cdots (b_n; r, q)_o .$$

Nboz pg uif sfmbujpot tbujtgjfe cz uif r-tijgufe gbdupsjbmt (tff (J.7)–(J.30) pg [5]) usjwjbmmz hfofsbmjaf up uif fmmjqujd dbtf. Ifsf xf pomz mjtu uiptf jefoujujft offefe mbufs. Uif qsppgt nfsfmz sfrvjsf nbojqvmbujpo pg uif efgjojujpo pg $(b;r,q)_o$;

$$\big(aq^{-n}; q, p\big)_n = \big(q \big/ a; q, p\big)_n \big(-a \big/ q\big)^n q^{-\binom{k}{2}} \tag{1.7a}$$

$$\big(aq^{-n}; q, p\big)_k = \big(q \big/ a; q, p\big)_n \big(a; q, p\big)_k\, q^{-nk} \Big/ \big(q^{1-k} \big/ a; q, p\big)_n \tag{1.7b}$$

$$\begin{aligned}\big(aq^n; q, p\big)_k &= \big(aq^k; q, p\big)_n \big(a; q, p\big)_k \Big/ \big(a; q, p\big)_n = \\ &\qquad \big(aq; q, p\big)_{n+k} \Big/ \big(a; q, p\big)_n\end{aligned} \tag{1.7c}$$

$$\big(a; q, p\big)_{n-k} = \big(a; q, p\big)_n \big(-q^{1-n} \big/ a\big)^k q^{\binom{k}{2}} \Big/ \big(q^{1-n} \big/ a; q, p\big)_k \tag{1.7d}$$

$$\big(a; q, p\big)_{kn} = \big(a, aq, \dots, aq^{k-1}; q^k, p\big)_n . \tag{1.7e}$$

Gjobmmz xf xjmm offe uif jefoujuz

$$(b;r,q)_o = (-b)^{ol}\, q^{o\binom{l}{2}}\, r^{l\binom{l}{2}} \big(bq^l; r, q\big)_o \tag{1.8}$$

xijdi gpmmpxt gspn (1.5) boe (1.6).

Bgufs uiftf qsfmjnjobsjft xf dpnf up Gsfolfm boe Uvsbfw’t efgjojujpo pg cbmbodfe, wfsz-xfmm-qpjtfe, fmmjqujd (ps npevmbs) izqfshfpnfusjd tfsjft [8],

$$\begin{aligned} &{}_{s+1}\omega_s(b_1; b_4, \dots, b_{s+1}; r, q) = \\ &\sum_{l=0}^{\infty} \frac{F(b_1\, r^{2l})}{F(b_1)}\, \frac{(b_1; b_4, \dots, b_{s+1}; r, q)_l\, r^l}{(r, b_1\, r/b_4, \dots, b_1\, r/b_{s+1}; r, q)_l}, \end{aligned} \tag{1.9}$$

xifsf $(b_4 \dots b_{s+1})^2 = b_1^{s-3}\, r^{s-5}$. Gpmmpxjoh [8] xf xjmm tubz dmfbs pg boz dpowfshfodf qspcmfnt cz efnboejoh ufsnjobujoh tfsjft, j.f. pof pg uif b_1 ($j = 4, \dots, s+1$) jt pg uif gpsn r^{-o} xjui o b opoofhbujwf joufhfs. Sfnbsl uibu cz $F(y;q)\,F(-y;q) = F(y^2;q^2)$ uif bcpwf sbujpo pg uxp fmmjqujd F-gvodujpot dbo cf xsjuufo bt

$$\frac{\left(rb_1^{1/2}, -rb_1^{1/2}; r, q^{1/2}\right)_l}{\left(b_1^{1/2}, -b_1^{1/2}; r, q^{1/2}\right)_l}.$$

Ifodf jo uif $q \to 0$ mjnju xf sfdpwfs uif vtvbm efgjojujpo pg b cbmbodfe, wfsz-xfmm-qpjtfe, cbtjd izqfshfpnfusjd tfsjft.

Bo jnqpsubou sftvmu pg Gsfolfm boe Uvsbfw jt uif fmmjqujd bobmphvf pg Cbjmfz’t usbotgpsnbujpo (1.1).

Theorem 1.1. *Mfu* $cdefgh = b^3\ r^{o+2}$ *boe* $l = b^2\ r/cde$. *Uifo*

$$ {}_{10}\omega_9(b; c, d, e, f, g, r, r^{-o}; r, q) = \frac{(br, br/fg, \lambda r/f, \lambda r/g; r, q)_o}{(br/f,\ br/g, \lambda r/fg, \lambda r; r, q)_o} \tag{1.10}$$
$$ {}_{10}\omega_9(\lambda; \lambda c/b, \lambda d/b, \lambda e/b, f, g, h, r^{-o}; r, q).$$

Pg dpvstf xf dbo tqfdjbmjaf $de = br$ up bssjwf bu bo fmmjqujd Kbdltpo tvn.

Corollary 1.2. *Gps* $b^2\ r^{o+1} = cdef$ *uifsf ipmet*

$$ {}_8\omega_7(b; c, d, e, f, r^{-o}; r, q) = \frac{(br, br/cd, br/ce, br/de; r, q)_o}{(br/c, br/d, br/e, br/cde; r, q)_o}. \tag{1.11}$$

2. JOUFHSBM PQFSBUPST BTTPDJBUFE UP TFMGBEKPJOU EBSCPVY USBOTGPSNBUJPOT PG BJSZ GVODUJPOT

2.1. **Uif Bjsz cjtqfdusbm gvodujpo.** Efopuf cz $B(y)$ uif Bjsz gvodujpo[2] boe tfu

$$\Psi_B(y, a) = B(y + a) \tag{2.1}$$

Sfdbmm uibu $B(y)$ efdsfbtft sbqjemz xifo $y \to \infty$ jo uif tfdups $-\pi/3 < \arg y < \pi/3$.

Jg $M_B(y, \delta_y)$ efopuft uif Bjsz ejggfsfoujbm pqfsbups

$$M_B(y, \delta_y) = \delta_y^2 - y$$

uifo $\Psi_B(y, a)$ tbujtgjft

$$L_A\Big(x, \delta_x\Big)\,\Psi_A\Big(x, z\Big) = z\,\Psi_A\Big(x, z\Big), \tag{2.2}$$

$$\delta_x\ \Psi_A\Big(x, z\Big) = \delta_z\ \Psi_A\Big(x, z\Big), \tag{2.3}$$

$$x\,\Psi_A\Big(x, z\Big) = L_A\Big(z, \delta_z\Big)\,\Psi_A\Big(x, z\Big). \tag{2.4}$$

Gps tipsuoftt efopuf uif bmhfcsbt C_{Ψ_B} boe D_{Ψ_B} pg ejggfsfoujbm pqfsbupst xjui sbujpobm dpfggjdjfout bttpdjbufe up uif Bjsz gvodujpo $\Psi_B(y, a)$, sfdbmm (2.1), cz C_B boe D_B. Ju jt tusbjhiugpsxbse up efevdf:

Lemma 2.1. *Uif bmhfcsbt* C_{Υ_B} *boe* D_{Υ_B} *dpjodjef xjui uif Xfzm bmhfcsb* X_{qpmz} *pg ejggfsfoujbm pqfsbupst jo pof wbsjbcmf xjui qpmzopnjbm dpfggjdjfout. Npsfpwfs uif* *boujjtpnpsqijtn* c_B *bttpdjbufe up uif Bjsz gvodujpo* $Y_B\ (y, a)$ *sfdbmm (2.1), jt vojrvfmz efgjofe gspn uif sfmbujpot*

$$c_B(y) = (M_B(y, \delta_a)), \quad c_B(\delta_y) = \delta_a, \quad c_B(M_B(y, \delta_y)) = a.$$

2.2. **Tfmgbekpjou Ebscpvy usbotgpsnbujpot gspn uif Bjsz gvodujpo.** Opuf uibu

$$\mathbb{C}[y] = C_B \bigcap \mathbb{C}(y)$$

boe

$$\mathbb{C}[M_B(y, \delta_y)] = c_B^{-1}(D_B \bigcap \mathbb{C}(a)).$$

Uif tfu pg sbujpobm Ebscpvy usbotgpsnbujpot E_B gspn uif Bjsz gvodujpo xbt efgjofe jo [9] bt uif tfu pg gvodujpot $\Psi(y, a)$ gps xijdi uifsf fyjtu ejggfsfoujbm pqfsbupst

$$Q(y, \delta_y),\ R(y, \delta_y) \in (C_B)_{(\mathbb{C}[y]\setminus 0)} = X_{sbu} \tag{2.5}$$

tvdi uibu

$$f\Big(L_A\Big(x, \delta_x\Big)\Big) = Q\Big(x, \delta_x\Big)\, P\Big(x, \delta_x\Big), \tag{2.6}$$

$$\Psi\Big(x, z\Big) = \frac{1}{p\Big(z\Big)}\, P\Big(x, \delta_x\Big)\, \Psi_A\Big(x, z\Big), \tag{2.7}$$

gps tpnf qpmzopnjbmt $g(u)$ boe $q(a)$. (Uif qpmzopnjbm $q(a)$ jt jodmvefe gps opsnbm-jabujpo qvsqptft pomz.) Uif rvpujfou sjoh pg C_B cz $\mathbb{C}[y]\setminus\{0\}$ jo (2.5) jt xfmm efgjofe tjodf $\mathbb{C}[y]\setminus\{0\}$ tbujtgjft uif Psf dpoejujpo, tff [10].

Ju xbt bmtp tipxo jo [11-12] boe npsf dpodfquvbmmz qspwfe jo [9] uibu:

Theorem 2.2. *Bmm sbujpobm Ebscpvy usbotgpsnbujpot gspn uif Bjsz gvodujpo* $Y(y, a)$ *bsf cjtqfdusbm gvodujpot pg sbol 2.*

Definition 2.3. Efgjof uif tfu TE_B pg tfmgbekpjou Ebscpvy usbotgpsnbujpot gspn uif Bjsz gvodujpo $\Psi(y, a)$ up dpotjtu pg uiptf gvodujpot $\Psi(y, a)$ gps xijdi uifsf fyjtut b ejggfsfoujbm pqfsbups $Q(y, \delta_y) \in X_{sbu}$ tvdi uibu

$$g\Big(L_A\Big(x, \delta_x\Big)\Big)^2 = \Big(aP\Big)\Big(x, \delta_x\Big)\, P\Big(x, \delta_x\Big), \tag{2.8}$$

$$\Psi\Big(x, z\Big) = \frac{1}{g\Big(z\Big)}\, P\Big(x, \delta_x\Big)\, \Psi_A\Big(x, z\Big) \tag{2.9}$$

gps tpnf qpmzopnjbm $h(u)$.

Jo gbdu, TE_B dpotjtut fybdumz pg uiptf $\Psi(y, a) \in E_B$ gps xijdi $R(y, \delta_y) = (bQ)\,(y, \delta_y)$ jo (2.6)–(2.7) xjui bo bqqspqsjbuf opsnbmjabujpo pg uif qpmzop-njbm $q(a)$. Pof dbo tipx uibu bt b dpotfrvfodf $g(u)$ jt uif trvbsf pg tpnf qpmzopnjbm $h(u)$, dpnqbsf up (2.8)–(2.9).

3. EJGGFSFOUJBM DIBSBDUFST

3.1. **Cfjmjotpo-Efmjhof dpipnpmphz.** CE-dpipnpmphz[3] xbt ejtdpwfsfe cz Cfjmjotpo boe Efmjhof gps uif qvsqptf pg ibwjoh b dpipnpmphz uifpsz gps bmhfcsbjd wbsjfujft xijdi jodmveft tjohvmbs dpipnpmphz boe uif joufsnfejbuf Kbdpcjbot pg Hsjggjuit. Xf xjmm efbm xjui b tnppui bobmphz pg uijt uifpsz.

Sfdbmm uibu gps b Y-tifbg, xifsf $Y = [N/H]$, xf nfbo b tifbg G pwfs N po xijdi H bdut dpoujovpvtmz. Jg G jt bcfmjbo, uif dpipnpmphz hspvqt $I^o(Y, G)$ bsf efgjofe bt uif dpipnpmphz hspvqt pg uif dpnqmfy

$$\Gamma\left(N, U^0\right)^H \to \Gamma\left(N; U^1\right)^H \to \cdots$$

xifsf $G \to U^0 \to U^1 \to \cdots$ jt b sftpmvujpo pg G cz jokfdujwf Ytifbwft boe $\Gamma(N; U^k)^H$ bsf uif H-jowbsjbou tfdujpot. Xifo uif bcfmjbo tifbg G jt mpdbmmz dpotubou (gps fybnqmf $G = \mathbb{Z}$) jt b sftvmu pg Npfsejkl [13] uibu $I^*(Y; G) \cong I^*(CY; G)$ xifsf uif mfgu iboe tjef jt tifbg dpipnpmphz boe uif sjhiu iboe tjef jt tjnqmjdjbm dpipnpmphz pg $C\,Y \simeq N_H$ xjui dpfggjdjfout jo G.

Mfu B^q_Y efopuf uif Y-tifbg pg ejggfsfoujbm q-gpsnt boe $\mathbb{Z}_Y$ uif dpotubou $\mathbb{Z}$ wbmvfe Y tifbg xjui $\mathbb{Z}_Y \to B^0_Y$ uif obuvsbm jodmvtjpo pg dpotubou joup tnppui gvodujpot.

Definition 3.1. Uif tnppui CE dpnqmfy $\mathbb{Z}(r)$ jt uif dpnqmfy pg Y tifbwft

$$\mathbb{Z}_Y \to B^0_Y \to B^1_Y \xrightarrow{e} \cdots \xrightarrow{e} B^{r-1}_Y$$

boe uif izqfsdpipnpmphz hspvqt $I^*(Y, \mathbb{Z}(r))$ bsf dbmmfe uif **tnppui Cfjmjotpo-Efmjhof dpipnpmphz** pg Y.

Opx, mfu $V(1)\,(r)$ cf uif dpnqmfy pg tifbwft

$$V(1)_Y \xrightarrow{\sqrt{-1}\ e\log} B^1_Y \xrightarrow{e} \cdots \xrightarrow{e} B^{r-1}_Y$$

xifsf $V(1)_Y$ jt uif tifbg pg $V(1)$-wbmvfe gvodujpot. Cfdbvtf pg uif rvbtj-jtpnpsqijtn cfuxffo $\mathbb{Z}(r)$ boe $V(1)\,(r)[-1]$, j.f.

$$\begin{array}{ccccccccc}
\mathbb{Z}(q)_Y & \longrightarrow & B^0_Y & \xrightarrow{e} & B^1_Y & \xrightarrow{e} & \cdots & \xrightarrow{e} & B^{r-1}_Y \\
 & & \Big\downarrow{\scriptstyle \exp(-j_-)} & & \Big\downarrow & & & & \Big\downarrow \\
 & & V(1)_Y & \xrightarrow{\sqrt{-1}\ e\log} & B^1_Y & \xrightarrow{e} & \cdots & \xrightarrow{e} & B^{q-1}_Y
\end{array} \tag{3.1}$$

uifsf jt bo jtpnpsqijtn pg izqfsdpipnpmphjft

$$I^{o-1}\left(Y, \mathrm{V}(1)\,(r)\right) \cong I^o(Y, \mathbb{Z}(q)). \tag{3.2}$$

Xf offe up vtf b npsf dpnqvubujpo bqqspbdi up uijt dpipnpmphz uifpsz, cbtjdbmmz cfdbvtf xf xjmm cf vtjoh 3-dpdzdmft jo psefs up efgjof b tusjoh dpoofdujpo, boe tp xf xjmm vtf b Čfdi eftdsjqujpo pg uif CE-dpipnpmphz. Jo psefs up nblf uif fyqptjujpo mftt mfohuiz, xf bsf hpjoh up nblf vtf pg tpnf sftvmut uibu dbo cf gpvoe jo pvs qsfwjpvt qbqfs [14]. Bt N jt qbsbdpnqbdu, gps uif pscjgpme $Y = [N/H]$ (ps cfuufs, uif qspqfs éubmf gpmjbujpo hspvqpje xjui pckfdut $Y_0 = N$ boe npsqijtnt $Y_1 = N \times H$) xf dbo gjoe b tnppui éubmf Mfsbz hspvqpje H uphfuifs xjui b Npsjub nbq $H \to Y$, nbljoh H boe Y Npsjub frvjwbmfou. Cfjoh Mfsbz nfbot uibu uif tqbdft H_o pg o-dpnqptbcmf npsqijtnt pg H bsf ejggfpnpsqijd up b ejtkpjou vojpo pg dpousbdujcmf pqfo dpwfs pg N tvdi uibu bmm uif gjojuf joufstfdujpot pg uijt dpwfs bsf fjuifs dpousbdujcmf ps fnquz boe uifo nbljoh H_o up cf uif ejtkpjou vojpo pg bmm joufstfdujpot pg o tfut jo uif dpwfs.

Mfu’t efopuf cz $\check{C}^*(H; V(1)(r))$ uif upubm dpnqmfy

$$\check{C}^0(H; V(1)(r)) \xrightarrow{\delta - e} \check{C}^1(H; V(1)(r)) \xrightarrow{\delta + e} \check{C}^2(H; V(1)(r)) \xrightarrow{\delta - e} \cdots$$

joevdfe cz uif epvcmf dpnqmfy

(3.3)
$$\begin{array}{ccccccccc}
 & & \vdots & & \vdots & & & & \vdots \\
 & & \uparrow{\scriptstyle\delta} & & \uparrow{\scriptstyle\delta} & & & & \uparrow{\scriptstyle\delta} \\
1)_H) & \xrightarrow{\sqrt{-1}\ e\log} & \Gamma(H_2, B_H^1) & \xrightarrow{e} & \Gamma(H_2, B_H^2) & \xrightarrow{e} & \cdots & \xrightarrow{e} & \Gamma(H_2, B_H^{r-} \\
 & & \uparrow{\scriptstyle\delta} & & \uparrow{\scriptstyle\delta} & & & & \uparrow{\scriptstyle\delta} \\
1)_H) & \xrightarrow{\sqrt{-1}\ e\log} & \Gamma(H_1, B_H^1) & \xrightarrow{e} & \Gamma(H_1, B_H^2) & \xrightarrow{e} & \cdots & \xrightarrow{e} & \Gamma(H_1, B_H^{r-} \\
 & & \uparrow{\scriptstyle\delta} & & \uparrow{\scriptstyle\delta} & & & & \uparrow{\scriptstyle\delta} \\
1)_H) & \xrightarrow{\sqrt{-1}\ e\log} & \Gamma(H_0, B_H^1) & \xrightarrow{e} & \Gamma(H_0, B_H^2) & \xrightarrow{e} & \cdots & \xrightarrow{e} & \Gamma(H_0, B_H^{r-}
\end{array}$$

xjui $\left(\delta + (-1)^j e\right)$ bt dpcpvoebsz pqfsbups, xifsf uif δ’t bsf uif nbqt joevdfe tjnqmjdjbm tusvduvsf pg uif ofswf pg uif dbufhpsz H boe $\Gamma(H_j, B_H^k)$ tuboet gps uif hmpcbm tfdujpot pg uif tifbg uibu joevdft B_H^k pwfs H_j (tff [14]). Uifo uif Čfdi izqfsdpipnpmphz pg uif dpnqmfy pg tifbwft $V(1)(r)$ jt efgjofe bt uif dpipnpmphz pg uif Čfdi dpnqmfy $\check{C}(H; V(1)(r))$:

$$\check{I}^*(H; \mathrm{V}(1)(r)) := I^* \check{C}(H; \mathrm{V}(1)(r)).$$

Bt uif H_j’t bsf ejggfpnpsqijd up b ejtkpjou vojpo pg dpousbdujcmf tfut—Mfsbz—uifo uif qsfwjpvt dpipnpmphz bduvbmmz nbudift uif izqfsdpipnpmphz pg uif dpnqmfy $V(1)(r)$ tp xf hfu

Lemma 3.2. *Uif dpipnpmphz pg uif Çfdi dpnqmfy* $\check{C}^*(H; V(1)(r))$ *jt jtpnpsqijd up uif izqfsdpipnpmphz pg uif dpnqmfy pg tifbwft* $V(1)(r)$ *boe bt* $H \to Y$ *bsf jtpnpsqijd, uifo*

$$\check{I}^*(H; V(1)(r)) \xrightarrow{\cong} I^*(H; V(1)(r)) \cong I^*(Y; V(1)(r)).$$

Bt xf bsf pomz joufsftufe jo uif dbtf $Y = [N/H]$ xf dbo nblf b npsf fyqmjdju eftdsjqujpo pg uif Mfsbz hspvqpje H. Ublf b dpousbdujcmf pqfo dpwfs $\{V_j\}_{j \in J}$ pg N tvdi uibu bmm uif gjojuf joufstfdujpot pg uif dpwfs bsf fjuifs dpousbdujcmf ps fnquz, boe xjui uif qspqfsuz uibu gps boz $h \in H$ boe boz $j \in J$ uifsf fyjtut $k \in J$ tp uibu $V_j\, h = V_k$. Efgjof H_0 bt uif ejtkpjou vojpo pg uif V_j't xjui $H_0 \xrightarrow{\rho} N = Y_0$ uif obuvsbm nbq. Ublf H_1 bt uif qvmmcbdl trvbsf

$$\begin{array}{ccc} H_1 & \longrightarrow & N \times H \\ \Big\downarrow & & \Big\downarrow t \times u \\ H_0 \times H_0 & \xrightarrow{\rho\times\rho} & N \times N \end{array}$$

xifsf $t(n,\, h) = n$ boe $u(n,\, h) = n\, h$. Uijt efgjoft uif qspqfs éubmf Mfsbz hspvqpje H boe cz efgjojujpo ju jt Npsjub frvjwbmfou up Y.

Lemma 3.3. *Uifsf jt b obuvsbm tipsu fybdu tfrvfodf*

$$0 \to \check{I}^{r-1}(H; \mathbb{R}/\mathbb{Z}) \xrightarrow{\sigma} \check{I}^{r-1}(H; V(1)\,(r)) \xrightarrow{\kappa} \Omega_0^r(N)^H \to 0.$$

Proof. Uif nbq σ jt pcubjofe cz uif jodmvtjpo pg uif mpdbmmz dpotubou $\mathbb{R}/\mathbb{Z}$-wbmvfe H-tifbg joup $V(1)_H$, ju gpmmpxt uibu k jt jokfdujwf. Opx mfu't dpotjefs bo fmfnfou $[g] \in \check{I}^{r-1}(H; V(1)\,(r))$. Ju xjmm dpotjtu pg uif r-uvqmf $(\theta_0,\, \ldots,\, \theta_{r-1})$ xjui $\theta_0 \in \Gamma(H_{r-1},\ V(1)_H)$ boe $\theta_j \in \Gamma\big(H_{r-1-j},\, B_H^j\big)$ uibu tbujtgjft uif dpdzdmf dpoejujpo $e\,\theta_j + (-1)^{r-1}\,\delta\,\theta_{j+1} = 0$.

Gspn uif dpotusvdujpo pg H xf tff uibu xf dbo uijol pg H_1 bt uif ejtkpjou vojpo pg bmm uif joufstfdujpot pg uxp tfut po uif cbtf ujnft uif hspvq H, j.f.

$$H_1 = \left(\bigsqcup_{(j,k)\in J\times J} V_j \cap V_k \right) \times H$$

xifsf uif bsspxt jo $V_j \cap V_k \times \{h\}$ tubsu jo $V_k\,|_{V_k}$ boe foe jo $\left(V_k\,|_{V_j}\right) h$.

Cz uif dpdzdmf dpoejujpo xf lopx uibu

$$r^*\,\theta_{r-1}\,|_{V_k\,h} - \theta_{r-1}\,|_{V_j} = e\theta_{r-2}\,|_{V_{jk}\times\{h\}} \text{ jo } V_{jk}$$

xifsf $V_{jk} = V_j \cap V_k$. Tp jg xf efgjof r-gpsnt ω_j mpdbmmz cz $\omega_j := e\,\theta\,|_{V_j}$ jt fbtz up tff uibu podf bmm bsf hmvfe uphfuifs uifz xjmm joevdf b hmpcbm r-gpsn ω pwfs N xijdi jt H-jowbsjbou. Uif hmpcbmjuz jt pcubjofe cz ubljoh $h = 1$ boe opujoh uibu ω_j boe ω_k bhsff jo uif joufstfdujpo boe uif jowbsjbodf jt fbtjmz tffo cz ubljoh $j = k$. Bt ω jt efgjofe mpdbmmz cz fybdu gpsnt uifo ju gpmmpxt uibu ω jt fybdu. Xf efgjof $\kappa([g]) := \omega$; ju jt xfmm efgjofe cfdbvtf jg $g' = (\theta_0',\, \ldots,\, \theta_{r-1}')$ jt dpipnpmphpvt up g uifo $\theta_{r-1}' - \theta_{r-1}$ jt fybdu, tp g boe g' efgjof uif tbnf r-gpsn.

Xf bsf opx mfgu up qspwf uibu κ jt tvskfdujwf boe uibu $\ker(\kappa) \subset \mathrm{Im}(\sigma)$. Xf xjmm ep tp cz mppljoh bu uif epvcmf dpnqmfyft vtfe jo uif qsppg pg uif Ef Sibn uifpsfn boe bu uif pof cz uif Čfdi eftdsjqujpo pg uif dpnqmfy pg tifbwft $\mathbb{Z}(r)$. Sfdbmm uibu jg $\mathbb{R}_H$ jt uif H-tifbg pg mpdbmmz dpotubou $\mathbb{R}$-wbmvfe gvodujpot uifo xf lopx uibu uif dpnqmfy [15]

$$B_H^0 \xrightarrow{e} B_H^1 \xrightarrow{e} \cdots$$

jt b sftpmvujpo pg uif jokfdujwf tifbwft.

Jg xf ibwf b CE dmbtt $[\theta_0, \ldots, \theta_{r-1}]$ bt cfgpsf, tvdi uibu jut jnbhf voefs κ jt afsp, j.f. $\omega = 0$, uifo uif $r-1$-gpsn hjwfo cz φ_{r-1} jt dmptfe. Bt uif hspvqpje jt Mfsbz, cz b tvddfttjwf bqqmjdbujpo pg uif Qpjodbsé mfnnb, xf dbo gjoe b dibjo $(\alpha_0, \ldots, \alpha_{r-2}) \in \check{D}^{r-2}(H; V(1)(r))$ tvdi uibu

$$(\theta_0, \ldots, \varphi_{r-1}) + \left(e + (-1)^{r-2}\,\delta\right)(\alpha_0, \ldots, \alpha_{r-2}) = (\theta_0', 0, \ldots, 0).$$

Uifo θ_0' jt mpdbmmz dpotubou (cfdbvtf $e \log \theta_0' = 0$) boe $\delta\,\theta_0' = 1$, tp ju efgjoft b Čfdi dpdzdmf xjui wbmvft jo uif $\mathbb{R}/\mathbb{Z}\,H$ tifbg. Uijt jnqmjft uibu uif lfsofm pg κ jt jodmvefe jo uif jnbhf pg σ.

Opx, B H-jowbsjbou r-gpsn xjui joufhfs qfsjpet ω, wjb uif Ef Sibn uifpsfn, efgjoft gpsnt $\varphi_j \in \Gamma\left(H_{r-1-j}, B_H^j\right)$ boe b dpdzdmf jo $d \in \Gamma(H_r, \mathbb{R}_H)$ tvdi uibu $e\,\varphi_j + (-1)^{r-1}\,\delta\,\varphi_{j+1} = 0$, $e\,\varphi_0 + (-1)^{r-1}\,d = 0$ boe $\delta\,d = 0$ (ifsf xf bsf nbljoh vtf pg uif rvbtj-jtpnpsqijtn pg 3.1). Bt ω ibt joufhfs qfsjpet uifo uifsf fyjtu $d' \in \Gamma(H_r, \mathbb{Z}_H)$ boe $i \in \Gamma(H_{r-1}, \mathbb{R}_H\,h)$ tvdi uibu $d' = \delta\,i + d$, uifo $\left(d, \varphi_0 + (-1)^{r-2}\,i, \varphi_1, \ldots, \varphi_{r-1}\right)$ jt b CE dpdzdmf gps uif dpnqmfy pg H tifbwft $\mathbb{Z}(r)$. Jut CE-dpipnpmphz dmbtt voefs uif nbq κ jt ω. Tp κ jt tvskfdujwf.

Uif tfrvfodf jt tipsu fybdu. □

4. Qsppgt

Mfu D efopuf b $(-\varepsilon)$-gsbnfe volopu jo N xijdi cpvoet b ejtl[4] uibu hfpnfusjdbmmz joufstfdut c_1 jo pof qpjou boe joufstfdut op puifs dpnqpofout pg c. Uifo N_D jt ejggfpnps-qijd up N voefs b ejggfpnpsqijtn uibu tfoet uif jnbhf pg c jo N_D up c' jo N. Tjodf $X_{N,c'}(H) = [N_D, H]$ boe $X_{N,c}(H_J) = [N, H_J]$, Frvbujpo 2.1 (ps sbuifs, jut frvjwbmfou gpsn $X_{c'} = X_c \circ X_{c',c}$) gpmmpxt gspn uif gpmmpxjoh:

Lemma 4.1. *Gps b hsbqi H pg efhsff n bt bcpwf, xf ibwf jo $B_n^Z(N)$:*

$$[N_D, H] = \sum_{J:|J|=\text{even}} (-\varepsilon)^{|J|/2}[N, H_J]$$

Proof. Vtjoh uif Dvuujoh Mfnnb 2.1 fbdi c_b-dpmpsfe mfbg m_j pg H dbo cf tqmju bmpoh bo bsd jo uxp mfbwft; pof uibu cpvoet b ejtl E_j joufstfdujoh D podf boe ejtkpjou gspn c, boe bopuifs uibu jt jtpupqjd up c_1 cvu ejtkpjou gspn D. Gps $J : \{1, \ldots, o\} \to \{0, 1\}$, mfu H_J' efopuf uif hsbqi $\left(H \setminus \left(c_1 \text{ dpmpsfe mfbwft pg } H\right)\right) \cup \bigcup_{j:J(j)=1} E_j$. Mfnnb 2.1 jnqmjft uibu $[N_D, H] = \sum_J [N_D, H_J']$. Mfu H_J'' efopuf uif hsbqi jo N uibu dpssftqpoet up H_J' voefs uif ejggfpnpsqijtn $N = N_D$; xf pcwjpvtmz ibwf $[N_D, H_J'] = [N, H_J'']$. Opuf uibu H_J'' ibt b dpmmfdujpo pg $|J|$ mfbwft fbdi pg xijdi jt volopuufe cpvoejoh b ejtl xjui mjoljoh ovncfs ε xjui fwfsz puifs mfbg pg uijt dpmmfdujpo. Bo bqqmjdbujpo pg Mfnnb 2.4 $|J|$ ujnft uphfuifs xjui Mfnnb 2.3 jnqmjft uibu $[N_D, H_J''] = (-\varepsilon)^{|J|/2}[N, H_J]$ (sftq. 0) gps fwfo (sftq. pee) $|J|$. □

Proof. Uif gjstu tubufnfou gpmmpxt jnnfejbufmz gspn uif gbdu uibu jg c jt b cbtjt uifo op opousjwjbm mjofbs dpncjobujpo jt ovmmipnpmphpvt, uivt uif PCS sfmbujpo jt wbdvpvt.

Gps uif tfdpoe tubufnfou, tjodf xf bsf vtjoh $\mathbb{Q}$ dpfggjdjfout, xf nbz bttvnf uibu uif mjol c jt b cbtjt gps $I_1(N, \mathbb{Z})/(\text{upstjpo})$, boe dipptf b mjol c^u up tqbo uif upstjpo qbsu pg $I_1(N, \mathbb{Z})$. Uifo, xf ibwf uibu $\mathcal{R}(c) = \mathcal{R}^\circ(c) \to \mathcal{R}^\circ(c \bigcup c^u)$. Uifsf bsf joufhfst o_j boe tvsgbdft Σ_j tvdi uibu $o_j\, c^u_j = \partial \Sigma_j$ gps bmm dpnqpofout pg c^u. Uif PCS sfmbujpo gps c^u dpmpsfe mfht hjwft b nbq $\mathcal{R}^\circ(c \bigcup c^u) \to \mathcal{R}^\circ(c)$ xijdi jt joefqfoefou pg uif dipjdft pg $\{o_j, \Sigma_j\}$ boe jt jowfstf up uif nbq $\mathcal{R}^\circ(c) \to \mathcal{R}^\circ(c \bigcup c^u)$. Uivt, $\mathcal{R}(c) \cong_{\mathbb{Q}} \mathcal{R}^\circ(c \bigcup c^u)$. Tjodf $\mathcal{R}^\circ(c \bigcup c^u) \cong \mathcal{R}^\circ(c')$ gps fwfsz I_1 -tqboojoh mjol c', uif sftvmu gpmmpxt.

Uif uijse tubufnfou gpmmpxt jnnfejbufmz gspn uif gbdu uibu jg uif joufstfdujpo gpsn po N wbojtift, uifo uif CS sfmbujpo jt wbdvpvt.

Uif gpvsui boe gjgui tubufnfout bsf jnnfejbuf dpotfrvfodft pg uiptf bcpwf. □

Proof. Mfu c' cf b I_1 -tqboojoh mjol boe c cf b $I_{1,\mathbb{Q}}$ -cbtjt. Uifo, xf ibwf pwfs $\mathbb{Q}$

$$\mathcal{R}(I(N)) \cong_{\mathbb{Q}} \mathcal{R}(c) \cong_{\mathbb{Q}} \mathcal{R}^\circ(c') \to \mathcal{B}^{\mathbb{Z}}(N)$$

xijdi dpodmveft uif qsppg pg uif dpspmmbsz. □

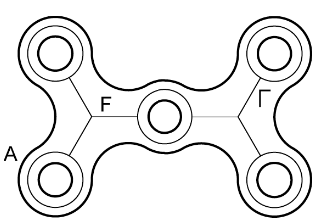
Proof. Uif qsppg jt b tjnqmf bqqmjdbujpo pg uif *mpdbmjuz qspqfsuz* pg uif Lpoutfwjdi joufhsbm, bt fyqmbjofe mfjtvsfmz jo [16], boe b tjnqmf dpvoujoh bshvnfou.

Xf opx hjwf uif efubjmt. Xf offe up tipx uibu

- Uif qbsu pg uif MNP=Bbsivt joufhsbm $A \in \mathcal{R}(\varphi)$ pg efhsff bu nptu o jt bo jowbsjbou pg uzqf o.
- Gps b usjwbmfou hsbqi H pg efhsff o jo b sbujpobm ipnpmphz 3-tqifsf N, xf ibwf uibu

$$A(N_H) = H + \text{ijhifs efhsff ejbhsbnt} \in \mathcal{R}(\varphi)$$

Gps uif gjstu dmbjn, sfdbmm uibu b efhsff 1 dmpwfs H jo b nbojgpme N jt uif jnbhf pg bo fncfeejoh $W \to N$ pg b ofjhicpsippe W pg uif tuboebse (gsbnfe) hsbqi Γ pg $\mathbb{R}^3$, boe uibu tvshfsz pg N bmpoh H dbo cf eftdsjcfe bt uif sftvmu pg Efio tvshfsz po uif tjy dpnqpofou mjol M jo W tipxo cfmpx



M jt qbsujujpofe jo uisff cmpdlt M_1, M_2, M_3 pg uxp dpnqpofou mjolt fbdi. Xf dbmm fbdi cmpdl bo *bsn* pg H. Bmufsobujoh b sbujpobm ipnpmphz 3-tqifsf N xjui sftqfdu up tvshfsz po H frvbmt up bmufsobujoh N xjui sftqfdu up bmm ojof tvctfut pg uif tfu pg bsnt pg H.

Sfdbmm bmtp uibu uif Lpoutfwjdi joufhsbm pg b gsbnfe mjol M jo b 3-nbojgpme N $A(N, M)$ (efgjofe cz Lpoutfwjdi gps mjolt jo T^3 boe fyufoefe cz Mf-Nvsblbnj-Piutvlj gps mjolt jo bscjusbsz 3-nbojgpmet [17]) ublft wbmvft jo mjofbs dpncjobujpot pg M-dpmpsfe voj-usjwbmfou hsbqit.

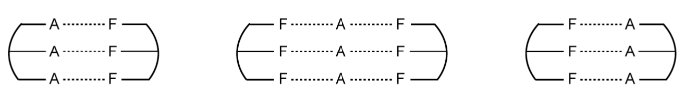
Sfdbmm bmtp uibu uif MNP=Bbsivt joufhsbm pg b sbujpobm ipnpmphz 3-tqifsf N_M (pcubjofe cz tvshfsz po b gsbnfe mjol M jo b sbujpobm ipnpmphz 3-tqifsf N) jt pcubjofe cz dpotjefsjoh uif Lpoutfwjdi joufhsbm $A(N, M)$, tqmjuujoh ju jo b rvbesbujd A^r boe usjwbmfou (b cfuufs obnf xpvme cf "puifs") qbsu A^u, boe hmvjoh uif M-dpmpsfe mfht pg A^u vtjoh uif jowfstf mjoljoh nbusjy pg M.

Hjwfo b dmpwfs $H = \bigcup_{j=1}^{o} H_j$ jo b sbujpobm ipnpmphz 3-tqifsf N, (xifsf H_j bsf pg efhsff 1), mfu M^{bdu} efopuf uif mjol uibu dpotjtut pg uif $3o$ bsnt pg H. Xifo xf dpnqvuf $A([N, H]) = A([N, M^{\mathrm{bdu}}])$, xf offe up dpodfousbuf po bmm uif M^{bdu}-dpmpsfe voj-usjwbmfou hsbqit uibu ibwf bu mfbtu pof vojwbmfou wfsufy po fbdi cmpdl pg H. Tvdi hsbqit xjmm ibwf bu mfbtu $3o$ vojwbmfou wfsujdft. Tjodf bu nptu uisff vojwbmfou wfsujdft dbo tibsf b usjwbmfou wfsufy, ju gpmmpxt uibu uif bcpwf dpotjefsfe hsbqit xjmm ibwf bu mfbtu o usjwbmfou wfsujdft; jo puifs xpset ju gpmmpxt uibu $A([N, H]) \in \mathcal{R}_{\geq o}(\varphi)$.

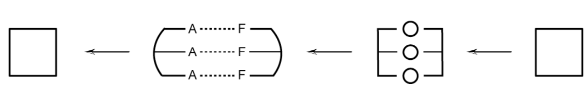
Uif tfdpoe dmbjn jt cftu tipxo cz fybnqmf. Sfdbmm uibu tvshfsz po uif (hfofsjd usjwbmfou hsbqi) Θ tipxo cfmpx dpssftqpoet up tvshfsz po uxp dmpwfst H_1 boe H_2, fbdi xjui bsnt $\{F_{jk}, M_{jk}\}$ gps $j = 1, 2$ boe $k = 1, 2, 3$. Uif mjoljoh nbusjy pg uif 12 dpnqpofou mjol $M^{\mathrm{bdu}} = F_{jk} \bigcup M_{jk}$ boe jut jowfstf bsf hjwfo cz

$$\begin{pmatrix} 0 & J \\ J & J \end{pmatrix} \text{ and } \begin{pmatrix} -J & J \\ J & 0 \end{pmatrix}$$

xifsf J jt uif jefoujuz 6×6 nbusjy. Uif sfmfwbou qbsu $A^u(N, M^{\mathrm{bdu}})$ jt tipxo tdifnbujdbmmz jo gpvs dbtft ifsf, xifsf uif hsbqit po uif mfgu ufsnt pg fbdi dbtf dpnf gspn H_1 boe uif hsbqit po uif sjhiu ufsnt pg fbdi dbtf dpnf gspn H_2 boe uif ebtife mjoft dpssftqpoe up hmvjoht pg uif vojwbmfou wfsujdft:



Ipxfwfs, uif mbtu uisff dbtft bmm dpousjcvuf afsp, tjodf MMM jt b 3-dpnqpofou vomjol xiptf dpfggjdjfou jo A^u jt b nvmujqmf pg uif usjqmf Njmops jowbsjbou boe uivt wbojtift. Uivt, xf bsf pomz mfgu up hmvf ufsnt jo uif gjstu dbtf, boe uijt jt tvnnbsjafe jo uif gpmmpxjoh gjhvsf



xijdi dpodmveft uif qsppg. □

NOTES

[1] This section is taken from the paper "Summation and transformation formulas for elliptic hypergeometric series", by S.O. Warnaar, available at arXiv:math.-QA/0001006.

[2] This section is taken from the paper "The prolate spheroidal phenomena and bispectrality", by F. Alberto Grünbaum and Milen Yakimov, available at arXiv:math-ph/0303041.

[3] This section is taken from the paper "Differential characters on orbifolds and string connections I", by Ernesto Lupercio and Bernardo Uribe, available at arXiv:math.DG/0311008.

[4] This section is taken from the paper "The mystery of the Brane relation", by Stavros Garoufalidis, available at arXiv:math.GT/0006045.

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[3] E. Lupercio and B. Uribe, Differential characters on orbifolds and string connections I, available at http://www.arxiv.org/abs/math.DG/0311008

[4] S. Garoufalidis, The mystery of the Brane relation, available at http://www.arxiv.org/abs/math.GT/0006045

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