Curvature Wavefront Sensing in Telescopes with Donut Fitting

Automatic real-time fine adjustment of a wide-field telescope using four defocused stars

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Project goal

Large ground-based telescopes require permanent fine adjustment of mirror position and shape on the micrometer/microrad accuracy scale. The goal is to control the VISTA telescope on Cerro Paranal, Chile, on the time scale of a few minutes.

Solution

We observe four stars, each defocused by ±1.2 mm. The images are ideally ring-shaped (“donuts”) and any aberrations yield information on the telescope misalignment state. Donut shape fitting is carried out in the Zernike polynomial basis, e.g. in 25 dimensions. In a second step, we invert the optical sensitivity matrix and multiply by the best fit Zernike vector to derive VISTA’s misalignment.

Result

The method is stable, highly accurate in testing and executes in a few seconds.

Project goal

ESO’s four-meter VISTA telescope is located on Cerro Paranal, situated in the Atacama desert in northern Chile at 2,500 meter altitude (Fig. 1). It can image an area on the sky with a diameter of about 2.5 degrees, equivalent to five times the size of the Full Moon. Such wide-field telescopes are employed to survey the sky night by night, looking for transient objects like comets and supernovae, but also to study more obscure physics like Dark Matter or quickly follow up discoveries by other telescopes.

The existing camera on VISTA will be replaced in 2023 by the multi-object fiber-fed spectrograph 4MOST that uses 2,400 optical fibers, each one moved by small actuators. The fibers relay the light of many different stars or galaxies to large spectrographs on the telescope backside that would not fit into the focal plane.

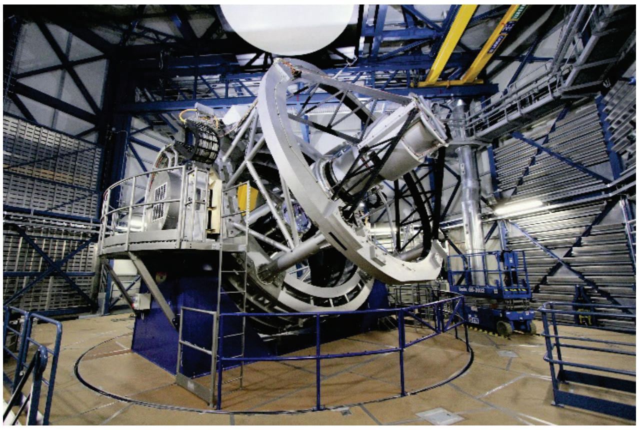

Fig. 1: VISTA telescope in its enclosure

Large telescopes require very precise mechanical alignment that needs to be updated every few minutes due to varying temperature and telescope inclination angles. To do so, VISTA has four “curvature wavefront sensing” cameras arranged around the rim of the astronomical focal surface. We image four different stars on those, but the cameras are slightly defocused by 1.2 mm above or below the focal plane. As a consequence, the cameras do not just see bright dots, but little pupil images. More precisely, each star is imaged as the annular region of the primary mirror not obscured by the secondary mirror M2 or its "spider" (four slim steel beams holding it in place).

It is our goal to maintain good telescope alignment by automatic "closed loop active optics" operation throughout the night. All the telescope operator has to do is validate that four preselected stars are properly acquired and imaged during each exposure (which can last up to 20 minutes each).

Solution

If the telescope is well aligned, the donuts look almost like circular annuli. Any misalignment manifests as a deformation/contortion that depends in a complicated manner on the camera location and the type of misalignment (e.g., M2 may be decentered and/or tilted by a very small amount against the optical axis), as shown in Fig. 2.

We choose to split the solution into two steps: In the first step, we fit each donut separately in the Zernike basis. The Zernike circle polynomials form an orthonormal basis on the unit disk and their radial part is readily provided by *Mathematica* (we have been asking Wolfram to also add the azimuthal part). Some of the low-order Zernike modes like defocus, coma or astigmatism are well known and, for instance, used in ophthalmology.

We employ a simplified raytracing scheme in which the ray landing positions in the donut are aberrated as controlled by Zernikes (center plot in Fig. 2). The lateral offsets are proportional to the Zernike derivatives in the focal plane which we first calculate analytically in *Mathematica* and then evaluate at the ray positions. This way, we combine *Mathematica's* strong analytical capabilities, in our case even in the complex domain, with numerics.

The objective function to be minimized equals the squared differences in the image brightness (right plot in Fig. 2), summed over all pixels. Before summing, we divide each square by the variance of the noise in the pixel, consisting mostly of shot noise that obeys a Poisson distribution. This "chi-squared" objective function is smooth and well behaved, but unfortunately the number of Zernikes to be fitted is rather high such as $n = 25$ or even $n = 34$. In the old VISTA instrument, the Nelder-Mead minimization algorithm was used which forms an n+1-dimensional simplex that converges to a local, and hopefully the global, function minimum. This method is robust but slow, limiting us to $n = 5$. In the new implementation, we employ the Levenberg-Marquardt (LM) algorithm instead, which is a mix of steepest descent (proceed in the direction of the strongest function decrease) and the Newton methods (quadratic function approximation near the minimum). We find that the LM method often converges within 4–6 steps, completing in under 2 seconds on a modern workstation. Moreover, the four donuts can be processed in parallel (`ParallelTable[]`). We are not aware of a single case in which the algorithm converged to a local minimum corresponding to an image obviously different from the simulation result.

In the second step, we have to map the Zernikes to telescope misalignments. The mathematical vehicle to represent this mapping is the optical sensitivity matrix of VISTA, evaluated at the four donut center positions. This matrix has been evaluated by the commercial optical simulation software *ZEMAX*, driven through a .NET API. The driver application *Sensitizer* has been written by ESO engineers and essentially disturbs the telescope prescription in each degree of freedom in turn by very small amounts while recording small variations in the Zernike aberrations.

The sensitivity matrix is pre-computed, its elements are interpolated in azimuth by the first three Fourier terms and, during telescope operation, evaluated numerically at each donut position. Finally, the four matrices are stacked up to be joined into a single matrix, inverted and multiplied by the stacked vector of the best fit Zernikes (e.g., of length 4×25 = 100).

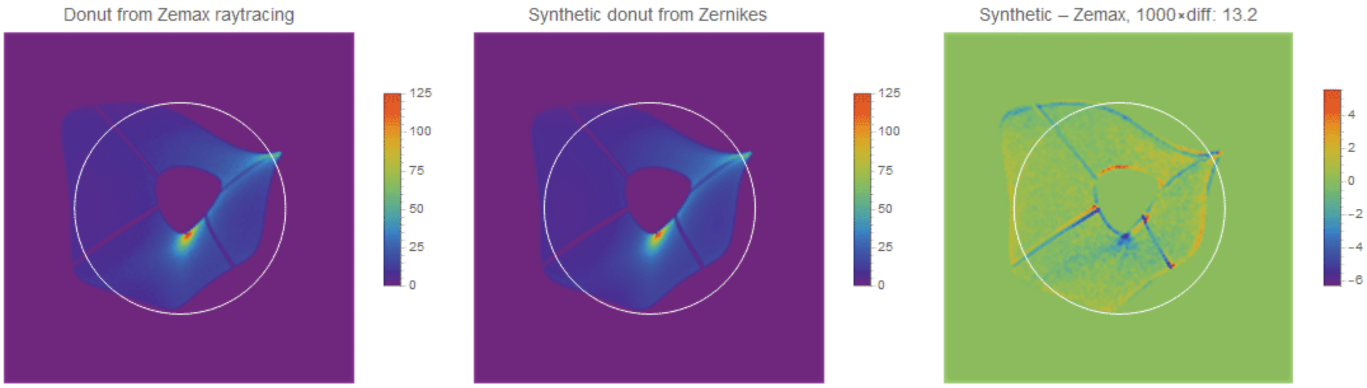

Fig. 2: Left: Donut of misaligned telescope simulated by Zemax, center: numerically approximated donut, right: difference An animation of the alignment cycle can be found here.

The described method works well, but we go one step further and apply the principle of maximum likelihood, in this context specifically known as Generalized Least Squares. Indeed, the LM algorithm not only provides an estimate of where the function minimum is, but also the fitting uncertainty. Zernike polynomials tend to steepen as their order grows which reduces their fitting error. Combined with the fact that the relative impact of noise decreases with the number of detected photons in each detector pixel, we can easily estimate the variance-covariance matrix of the Zernikes. This matrix is then used to weight the inversion of the sensitivity matrix. In mathematical terms, we weight the least-squares problem, which is an overdetermined equation system that becomes slightly contradictory due to measurement errors, in order to account for the shot noise statistics and arrive at the maximum likelihood perturbation. This weighting is particularly beneficial if the four stars have very unequal brightness (dim star ➔ larger relative errors). Finally, we also derive the uncertainty in each telescope perturbation degree of freedom.

There a few more subtleties that we cannot describe here. One example is the so-called seeing, namely the influence of atmospheric turbulence in the air column above the telescope, which blurs the star images (not shown in Fig. 2 for clarity). The degree of blurring strongly varies with location, time and observed wavelength, but often is of the order of 0.7 arcseconds on Cerro Paranal. We fit the seeing using convolution with a Moffat function kernel (aided by the Fast Fourier Transform), which adds one scalar degree of freedom to the LM minimization. Moreover, having four donuts located around the focal surface provides us with the luxury of some redundancy and we can actually solve the inversion problem with only three donuts. The latter is very desirable in case one of the images is garbled for some reason, polluted by a bright background object, or simply too dim. We have implemented a heuristic to detect such cases and automatically ignore deficient donuts.

Result

We have developed and implemented the algorithm in Wolfram *Mathematica* version 12.2. The development process is rapid, thanks to the strong graphical and analytical abilities. We also benefitted from previous work at ESO such as *Sensitizer*. While linearization schemes like the present one have been employed for many decades now, the detailed maths and physics of telescope optics are not trivial and require copious testing. We chose a step-by-step approach in which we first tested the optics basics like the simplified raytracing (actually, implemented just as a matrix-vector product) and then grew the complexity by combining subroutines. Certain low-level time-critical operations like counting rays on a virtual detector were compiled to the C language using the `Compile[]` statement. The LM algorithm is accessible by an option offered by *Mathematica's* `FindMinimum[]` command and flexible enough to allow for strong user customization of which we widely take advantage (e.g., we provide a custom numerical function Jacobian).

As a result, we expect to automatically and accurately control the VISTA alignment throughout every night of operation. We are looking forward to the “first light” of VISTA with the new instrument 4MOST in 2023. Stay tuned!

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